Int4Sci:
Scilab Interface for Interval Analysis

http://www-sop.inria.fr/coprin/logiciels/Int4Sci/

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Team Coprin: Robotics and...

Parallel Manipulator

ESRF, Accurate positioning manipulator

CMW LGV milling machine
Crane based on a cable driven robot

Crane rescue
Wrench-Feasible Workspace of Parallel Cable Driven Mechanisms

- $X$, position and orientation ($n$ dof)
  the interval vector (box) $[X] = [\underline{X}, \overline{X}]$
  a set of poses $\{X | \underline{X} \leq X \leq \overline{X}\}$

- $\tau_i$, Tension in $i = 1, \ldots, m$ cables
  the box $[\tau]$ Set of allowed tensions in the cables:
  $\{\tau \mid \tau_i \in [\tau_{imin} > 0, \tau_{imax}], \forall 1 \leq i \leq m\}$

- $f$, Total wrench exerted by the cables at $C$
  the box $[f]$ the set of wrenches (forces and moments)
  to be applied by the cables on the mobile platform

Static relation

$$W_{n \times m} \tau = f, \text{ with the wrench matrix } W(X) = -J^{-T}(X)$$
The Wrench-Feasible Workspace (WFW) is the set of robot poses for which the cable tensions allow to generated a given set of wrenches.
**Wrench-Feasible Workspace of Parallel Wire Driven Mechanisms**

**Algo:** Branch and prune on $[X]$
- Initial box $[X]_1$

for $\mathcal{X} = [X]_1, \ldots, [X]_i, \ldots$

- if $[X] \cap WFW = \emptyset$, Outer box
- if $[X] \subset WFW$, Inner box
- else bisection of $[X] = [X_{inf}] \cup [X_{sup}]$
  $\mathcal{X} \leftarrow \mathcal{X}, [X_{inf}], [X_{sup}]$
Wrench-Feasible Workspace of Parallel Wire Driven Mechanisms

Outer Boxes

\[ X \] lies fully outside the WFW if sufficient conditions (but not necessary)
First Test: \[ f \not\in [W][\tau] \];
Stronger Test: \( \forall \tau \in [\tau], W_\tau \neq f_y \) with \( f_y \) the set of vertices of the box \[ f \]

Consistency techniques

Inner Boxes …
Inner Boxes

$[X]$ lies fully inside the WFW if a sufficient condition (but not necessary)

$\forall W \in [W], \forall f \in [f], \exists \tau \in [\tau]$ such that $W\tau = f$

[Rohn2002]

The system of interval linear equations $[W]\tau = [f]$ is strongly feasible if and only if the $2^n$ systems of linear equations $W_y\tau = f_y$ are feasible

$W_y, f_y$ are a selection of $2^n$ extreme systems ($W_{y_{i,j}} = W_{i,j} \lor W_{i,j}$; $f_{y_i} = f_i$ or $f_i$)

Feasibility of linear systems:
Linear Programing approach

$\min c^T \tau$ subject to $W_y\tau = f_y, \tau \in [\tau]$ (with $c = 0$)
Examples: WFW

Planar 3-DOF Mechanism

- Tensions \([T_1, \ldots, T_4] = [1, 54] \text{ (N)}\)
- \([f] = \begin{bmatrix} -10, 10, -10, 10, -0.5, 0.5 \end{bmatrix}^T \text{ (N,N,N.m)}\)
- \(\phi = 0^\circ, \phi = 60^\circ\)
Examples: WFW

6-DOF parallel Mechanism driven by 8 wires

- Mobile platform: regular tetrahedron 0.1 m
- $[\tau] = [1, 540]$ (N)
- $[f] = \begin{bmatrix} [-10, 10]^T_{3 \times 1} & [-0.5, 0.5]^T_{3 \times 1} \end{bmatrix}^T$ (N), (N.m)
- $\forall$ euler’s angles $\left[-\frac{\pi}{12}, \frac{\pi}{12}\right], \left[-\frac{\pi}{12}, \frac{\pi}{12}\right], [0, 0]$ (rad)

Full WFW in time = 970 s

Box $[0.7, 1.3][0.4, 0.8][0.6, 1.3]$ (m)
guaranteed in the WFW (41 s)
Examples: Geometrical workspace of a flexible 3RRR planar robot

\[ GW \equiv \{ X \mid IK(X) \in [\theta_i, \bar{\theta_i}], \det(W(X)) \neq 0 \} \]

**Static relation**  
\[ W(X)_{n \times n}^T = f \]

Basic computation of the determinant bounds

Using ALIAS Library test on test regularity
To promote Interval Techniques in robotics

Interval analysis is interested in robotics

Our contributions

• Robot Kinematics

• Verification of robot performances:
  Geometrical workspace, Determination of robot Singularities,
  Accuracy, Statics capabilities

• Design (taken into account mechanical tolerances)

• Calibration, Control …

Robotics problems

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<th>Collaborations</th>
<th>Standard Tools</th>
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Team Coprin: Robotics and Interval Techniques

Origin of specialist members:
- 2 in robotics,
- 2 in constraints programming,
- 2 in applied mathematics

1) Develop Interval Techniques (Interval Analysis + Constraint programming)
2) Applied to robotics

Libraries
1999 ALIAS  www-sop.inria.fr/coprin/logiciels/ALIAS/ALIAS.html

Based on Bias/Profil IRC, Hamburg Univ. of Tech
http://www.ti3.tu-harburg.de/Software/PROFILEnglisch.html
User friendly toolkit for interval analysis methods diffusion

Aims
- To exemplify interval techniques
- To show the efficiency of these techniques
- To make them easily available

End-Users
- Students
- Non-experts in computer science
- Community researchers for prototyping

Wanted Properties
- Easy to install, to program
- To preserve the efficiency, IA capabilities
- To provide high level of IA functionalities
- To do not change user programming practices
- To do not reinvent the wheel

Interface for Interval Analysis
# Interface for Interval Analysis

Some solutions

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<th>Free</th>
<th>Symbolic</th>
<th>Interval Package</th>
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<td>Native Alias</td>
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Scilab Interface for Interval Analysis

- Matlab-like Open source
- Completeness functionalities (numerical tools, graphics) + add-ons
- Well known in academic and engineer community
- A quiet large number of users

- Supported by INRIA, R. Pereira engineer 2 years (2006-2007)

- Advanced type (overloading) $\rightarrow$ mlist
- Interface functionalities to create Interfaces to C, C++, Fortran
  
  *Toolbox compilation process* tbx_build_tools

Scilab progress …

Libraries linked

- Bias/Profil + Boost Interval tested in 2006
- Coprin advanced libraries : ALIAS
- others …
**Int4Sci**

2005 First prototype

**2007 Int4Sci 1.0 (R. Pereira, D. Daney)**

Interval type in Scilab \%interM  
Arithmetic package  
Interval linear solver package

Scilab 2.7  
Bias  
Profil 2.0.4

Windows, Linux, MacOS

**2010~11 Int4Sci 2.0 (D. Daney, B. Neveu)**

Interval type in Scilab \%interM  
Arithmetic package  
Interval linear solver package

Interval Polynomial package \%interP  
Interval Solver package

Scilab 5.2.2  
Bias  
Profil 2.0.8

Scilab 5.2.2 + Alias  
Alias

Windows, Linux, MacOS
Interval type in Scilab: mlist %interM

Scilab mlist: %interM

```scilab
x=mlist(["interM","dims","inf","sup","arith","mode"],
        size(a), a, b, "classical","double");
```

```scilab
--> z=interval(-2,5)
z =
| -2,5 |

--> M=#(rand(3,3),rand(3,3));
--> M(2:3,2:3)=[sum(M(2:3,:),"c") prod(M(2:3,:),"c")]
```

%interM_p(x)  
%interM_i_interM  
%interM_sum  
...
Int4Sci Interface

“Add” example

--> #(1,2) + #(-3,1)

c=%interM_s_interM(a,b)

[c.inf,c.sup,flag] = scioperator (a.inf,a.sup,'a',b.inf,b.sup);

Int intoperator (char *fname)

Int operate (const double *ainf ...)

VOID BiasAddII (BIASINTERVAL *...
Interval extended arithmetics

\[ t = (-1,2) + (-4,5) \]
\[ t = [-5,7] \]

\[ \log((-10,10)) \]
\[ \text{ans} = [-\infty, 2.3025851] \]

Interval Functions

\[ (-10,10):2 \]
\[ \text{ans} = \]
\[ \text{ans}(1) = [-10,0] \]
\[ \text{ans}(2) = [0,10] \]
Interval linear solver package

\[ A = \#([4,-1,1.5;-0.5,-7,1;-1.5,-0.7,2],[5,1,2.5;5,-5,2;-.5,-.5,3]) ; \\
\]

\[ b = \#([3;0;3],[4;2;4]) ; \\
\]

\[ x = A \backslash b ; \\
\]

\[ x = \text{l4Slinearsolve}(A,b,"GE") ; \\
\]

\[ x = \text{l4Slinearsolve}(A,b,"PHB") ; \\
\]

\[ x = \text{l4Slinearsolve}(A,b,x,"GS",10,0.89) ; \\
\]

\[ x = \]
Interval polynomials package

```latex
\texttt{-->p1 = intpoly([#(1,2),#(-5,2),#(0,2)])}

\texttt{p1 =}

\texttt{|-8,20|+|-18,12|*ix+|-6,4|*ix^2+1,1|*ix^3}

\texttt{-->p2 = intpoly([#(1,2),#(-5,2),#(0,2)],"x","coeff")}

\texttt{p2 =}

\texttt{|1,2|+|-5,2|*x+|0,2|*x^2}

\texttt{-->s = poly(0,'s');}

\texttt{-->a = [#(1,2)*s^5, #(1,2)+#(2)*s^2; 2+3*s,s^3]}

\texttt{a =}

\texttt{|1,2|*s^5} \hspace{2cm} \texttt{|1,2|+|2,2|*s^2}

\texttt{|2,2|+|3,3|*s} \hspace{2cm} \texttt{|1,1|*s^3}

\texttt{-->polyeval(a,[2,#(-%inf,2);#(1,2),0])}

\texttt{ans =}

\texttt{!|32,64| |-Inf,Inf| !}
\texttt{! ! !}
\texttt{!|5,8| |0,0| !}
```
Int4Sci 2.0
Installation

Install Libraries
- Scilab 5.2.2
- Profil/Bias (2.0.8)
  * Arithmetics
  * Intrv. Linear. Solve
- Alias
  * Intrv. Polynomial package

In Scilab

--> exec builder.sce
Int4Sci 2.0 loader

In Scilab

--> exec loader.sce

```
scilab-5.2.2

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Initialisation :
  Chargement de l'environnement de travail

--> exec loader.sce;
Start Toolbox int4sci
  Load macros
  Load gateways
  Load help

  Interval Arithmetic library is loaded
  Interval polynomial library is loaded

-->|
```
A small DEMO …
Challenges and “what is at stake”

✚ Increase high level functionalities
  → therefore users and developers interest for Int4Sci

✚ A perennial long lasting development and technical support Int4sci

  - Increase the contributions of external teams/projects

✚ Manage the dependency of Int4sci to external libraries

  - Ensure the good functioning and linking of low level libraries
  - Interoperability and interchanging of low level libraries

✚ Interface Ergonomics
Conclusion

- **Int4Sci**: a Scilab interface for interval Analysis
- **End-Users**: Students and Non experts (engineers, researchers)
- **Aims**: get a simple access to
  - basic functionalities
  - Link *your* advanced functionalities,
  - Prototyping

Road map

- release 2.0 (improve installation and Make binaries available)
- **Contribute** ....