An envelope for errors in hydrological model output caused by sampling and averaging

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14th GAMM-IMACS International Symposium on Scientific Computing, Computer Arithmetic and Validated Numerics

Outline

1. Motivation
   - Models in hydrology
   - Problem statement
2. Effect of Error Envelope
   - A simple linear reservoir
   - A switch with hysteresis

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Types of systems.

- River basins (also known as catchments)
- Series of Reservoirs
- Villages and cities

Questions.

- Model to find risk of floods
- Model to find optimal operation strategy
- Model to avoid storm drain or sewer overflow

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Model calibration.

- Preferably automated
- Input averaged over time intervals
- Output at discrete points in time

Problem.

- Penalty function for optimization
- Model may not see details of input
- Parameters may have overlapping effects

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Example

- parking lot
- roof

The Functions.

- Precipitation intensity $p(t)$ (millimeters per hour)
- Dynamic storage $s(t)$ (depth in millimeters)
- run-off $q(t) = c_0 s(t)$ (millimeters per hour)

The model

$$\dot{s} = p(t) - c_0 R(s)$$

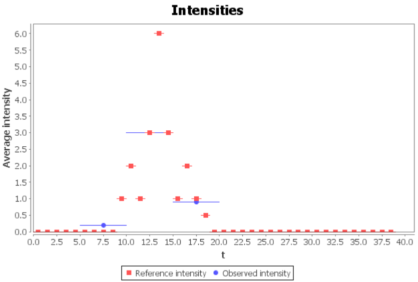
$$s(t_0) = s_0$$

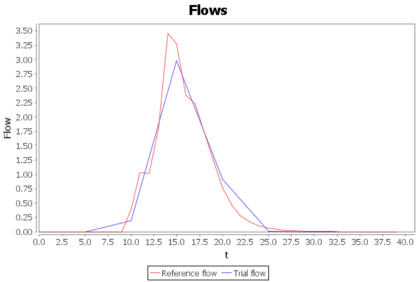
$$R(s) = \begin{cases} 0 & s \leq 0 \\ s & s > 0 \end{cases}$$

The observations.

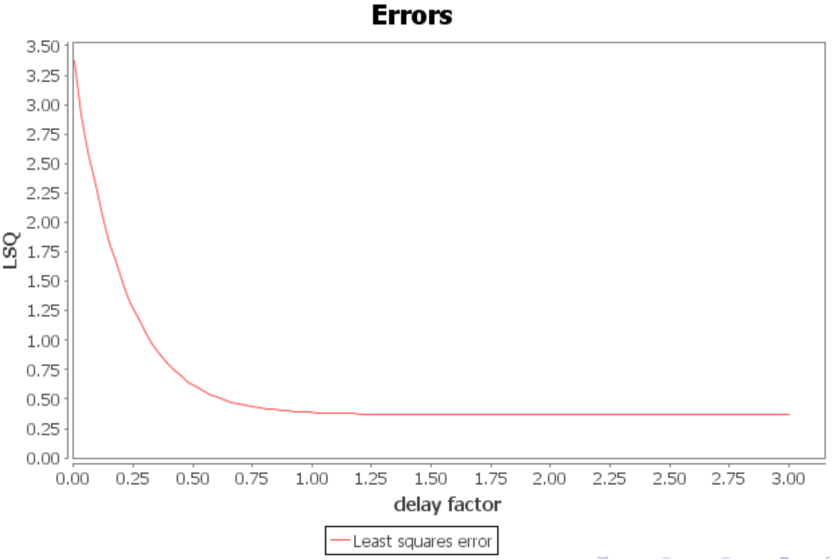
- Rain gauge data $P_k = \int_{t=t_k}^{t_{k+1}} p(t)\, dt$ (millimeters)
- Runoff data $q_k = q(t_k)$

Optimization without Intervals.

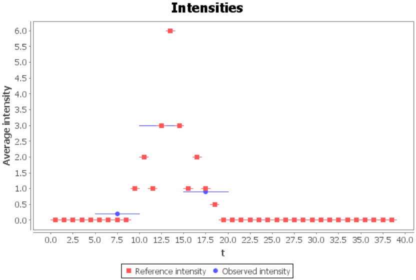


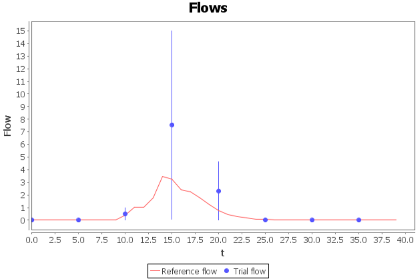


Least Squares Error.

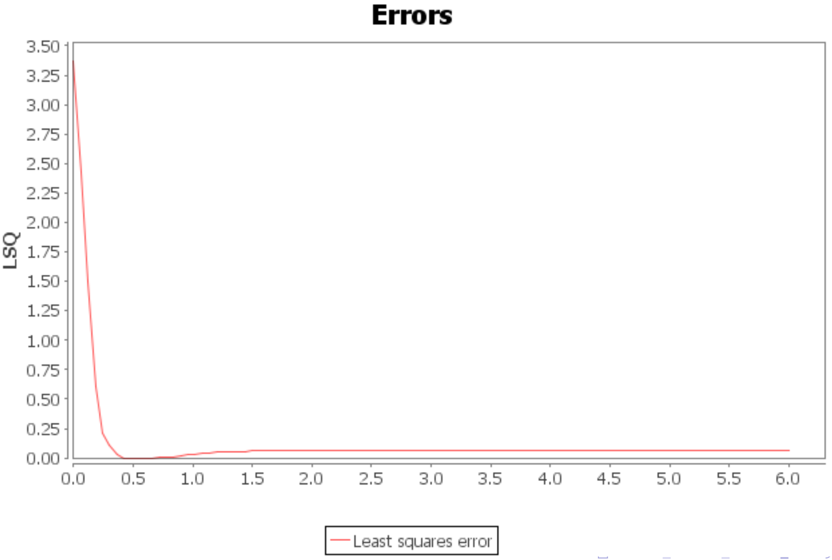


Optimization with Intervals.





Least Squares Error



Outline

Example

- Section of a sewer system that is emptied by a pump

The Functions.

- Precipitation intensity $p(t)$ (millimeters per hour)
- Dynamic storage $s(t)$ (depth in millimeters)
- run-off $q(t) = c_0 s(t)$ (millimeters per hour)

The model

$$\dot{s} = p(t) - c_0 u(s)$$

$$\dot{u} = H(1 - 2u(s))\,\delta(s - s_{\mathrm{on}}) - H(2u(s) - 1)\,\delta(s - s_{\mathrm{off}})$$

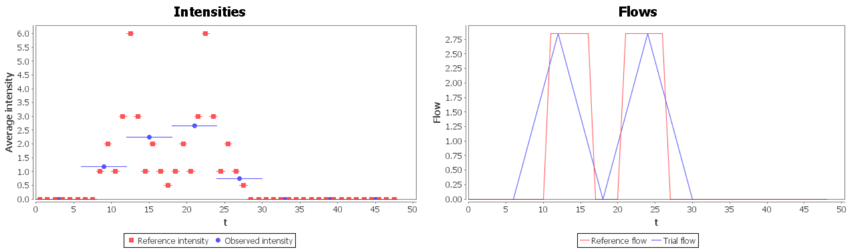
$$s(t_0) = s_0$$

$$H(s) = \begin{cases} 0 & s < 0 \\ 1 & s > 0 \end{cases}$$

The observations.

- Rain gauge data $P_k = \int_{t=t_k}^{t_{k+1}} p(t)\,dt$ (millimeters)
- Runoff data $q_k = q(t_k)$

Optimization without Intervals.



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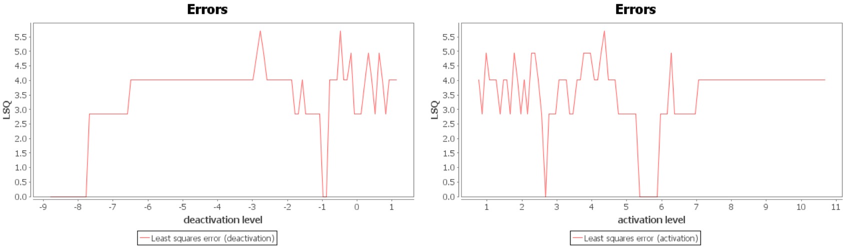
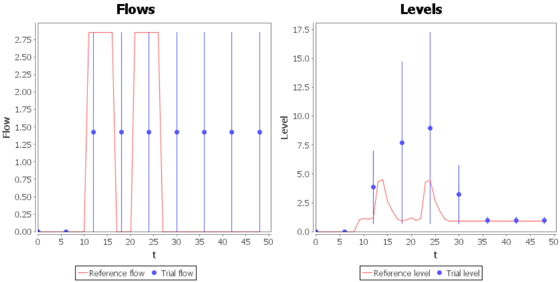


Optimization with Intervals.

- For simple inflows (one peak) this gets stuck in an indeterminate state
- Complex algorithm
- Error margins too large for simple inputs
- Problem: defined as on at $s(t) = s_{\text{on}}$, problem to get $s(t)$ below s_{on}
- Work in progress

Preliminary Results



Summary

- Even for simple problems averaging and sampling can mislead a computer.
- For systems with feedback interval calculations may help.

- Outlook
  - More complex components.
  - components combined in series or in parallel.